1.(10) Consider a continuous-time filter with impulse response \( h_c(t) \) and a system function \( H_c(s) = \frac{s + a}{(s + a + jb)(s + a - jb)} = \frac{\frac{1}{2}}{(s + a + jb)} + \frac{\frac{1}{2}}{(s + a - jb)} \) (note the partial fraction expansion has already been done). Use impulse invariance to determine \( H(z) \) for a discrete-time system using \( T_d = 1 \). Give simplified expressions for \( h_c(t) \), \( h[n] \), and \( H(z) \).

Recall

\[
\begin{align*}
A_k e^{s_k t} u(t) &\xrightarrow{L} s \text{ plane:} \quad \frac{A_k}{s-s_k} \xrightarrow{\text{Imp. Invar.}} z \text{ plane:} \quad \frac{T_d A_k}{1 - e^{s_k T_d} z^{-1}}
\end{align*}
\]

\[
h_c(t) = \frac{1}{2} \left( e^{-(a+jb)t} + e^{-(a-jb)t} \right) u(t) = e^{-at} \cos(bt) u(t)
\]

\[
h[n] = h_c(n T_d) = e^{-a n} \cos(b n) u(n)
\]

\[
H(z) = \sum_k \frac{T_d A_k}{1 - e^{s_k T_d} z^{-1}} = \frac{\frac{1}{2}}{1 - e^{-(a+jb)} z^{-1}} + \frac{\frac{1}{2}}{1 - e^{-(a-jb)} z^{-1}} = \frac{1 - e^{-a} \cos(b) z^{-1}}{1 - 2e^{-a} \cos(b) z^{-1} + e^{-2a} z^{-2}}
\]

2.(5) We wish to design FIR filters using the windowed filter design technique. Which of the following 3 window types would be most appropriate if you are designing a bandpass channel selection filter for a digital wireless communications receiver? The frequency range of \(-\pi \leq \omega \leq \pi\) is divided into many equal bandwidth channels, each of which is equally likely to contain interfering signals from other users. You want to select only your channel and reject all others. You must justify your response to receive credit for your answer.

i. Rectangular window.
ii. Dolph - Chebychev window, \( \alpha = 2.5 \)
iii. Tukey window, \( \alpha = 0.75 \)

- ii. Use the Dolph - Chebychev window which has equiripple sidelobe levels in the stopband. This minimizes the maximum sidelobe for uniform rejection of unwanted channels.