1. Enter all answers on the scantron "bubble" answer sheet.
2. Each problem is worth five points.
3. Unless otherwise requested, for numeric answers do not round your result. Enter the most significant digit into your bubble sheet.
4. Attach all your calculation worksheets to this test and return them to the testing center. Any requests of credit on incorrectly marked answers must be supported by a correct numerical answer on your calculation worksheets.
Questions 1 - 6 refer to the following figure of a magnitude frequency response for an optimal filter designed by the Parks-McClellan algorithm.

1. What type of generalized linear phase system is pictured?
   1) I; 2) II; 3) III; 4) IV; 5) V; 6) it is not generalized linear phase; 7) minimum phase; 8) multiband.

2. How many alternations are there?

3. What is the total filter length, \( N \), for the corresponding impulse response, \( h[n] \), i.e. how many filter taps are there?

4. Assume \( W(\omega) = 1.0 \) at \( \omega = 0.2\pi \), \( \delta_s = 0.21 \) and \( \delta_p = 0.07 \). What value of \( W(\omega) \) was used at \( \omega = 0.8\pi \)? Round to the nearest integer and bubble in the most significant digit.

5. Assume the filter was redesigned using the Parks-McClellan algorithm with all the same parameters except the passband corner frequency was moved to 0.75\( \pi \). What changes would you observe in the new frequency response, \( A(\omega) \)?
   a) no change;   b) a wider transition band;   c) one less alternation;
   d) increased passband ripple;   e) decreased passband ripple;
   f) increased stopband ripple;   g) decreased stopband ripple;   h) b, d, and f;
   i) b, e, and g;   j) d and g.

6. How many zeros are there on the unit circle for the corresponding Z transform?
Questions 7-9 refer to using the Kaiser-Bessel windowed filter method to design the following bandpass filter. The transition band corner frequencies (in radians) are $\omega_1 = \pi/2$, $\omega_2 = (5\pi)/8$ for the first transition, and $\omega_3 = (11\pi)/16$, $\omega_4 = (3\pi)/4$ for the second transition. The stopband ripple level should be below -53 dB.

7. Solve for the shape parameter $\beta$ that will meet this specification. Round to two significant digits and enter the second most significant.

8. Solve for the approximate filter length $N$. Enter the most significant digit.

9. When specifying the ideal desired impulse response, $h_d[n]$ to be windowed, what radian frequency, $\omega_s$, will you use for the left side edge of the second stopband? Calculate to at least four significant digits and enter the second most significant.

10. What window type would you choose for a bandpass filter design if your primary concern is rejecting a very strong interfering source whose frequency is known to be at least two octaves higher than your desired band?
   a) Hamming; b) Rectangle; c) Dolph-Chebychev $\alpha = 2.5$; d) Bohman; e) Gaussian; f) Triangle.

11. Suppose you have designed a bandpass filter using the windowed filter method and that its frequency response meets all of your specifications. In fact, you have significantly less ripple level than required in the passband and your second transition band is more narrow than specified. All other specifications are met just about exactly. You would like to keep your filter as short as possible to keep computations down. What should you do?
   a) Decrease filter length $N$ to exploit the excess room in transition band 2, since this will not affect the ripple levels.
   b) Change your window choice or change window parameter (e.g. $\beta$ in a Kaiser-Bessel window) since your window is too tapered, i.e. a different window would use the margin you have and allow passband ripple to increase within specification, while at the same time providing a more narrow main lobe/transition band. Thus you could then reduce filter length $N$ without exceeding transition band specs.
   c) Do both (a) and (b).
   d) Do neither (a) or (b) or else you will violate filter design specifications.
12. The discrete-time Fourier transform of each of the sequences plotted above has the same magnitude response. Match each sequence with its corresponding pole-zero plot. Which answer below matches all three correctly.
   a) I-α, II-β, III-γ;  b) I-β, II-γ, III-α;  c) I-γ, II-α, III-β;  d) I-β, II-α, III-γ;
   e) I-α, II-γ, III-β;  f) I-γ, II-β, III-α.

13. Which sequence above would be best as the impulse response for a filter system that you wished to have the least possible group delay?
   1) I;  2) II;  3) III.

14. Find $H(z)$ for sequence I expressed as a product of standard form first order terms, i.e. find the zeros (all amplitudes in the plot are exact multiples of 0.5). Express $H(z)$ as the product of an all pass system and a minimum phase system, i.e. $H(z) = H_{ap}(z) H_{min}(z)$. Enter the first significant digit of the zero location in $H_{ap}(z)$.

15. Which sequences correspond to generalized linear phase systems?
   a) I;  b) II;  c) III;  d) I and II;  e) I and III;  f) II and III;  
g) all of them;  h) none of them.
16. Consider a second order IIR discrete-time bandpass filter design based on the continuous-
time prototype \( H_c(s) = \frac{1}{s + a + jb} + \frac{1}{s + a - jb} \) using the bilinear transform method. Let 
\( T_d = 2.0 \). Which of the following is the resulting transfer function for the discrete-time filter?

a) \( H(z) = \frac{1 - z^{-1}}{1 + a + jb(1 - a - jb)z^{-1}} + \frac{1 - z^{-1}}{1 + a - jb(1 - a + jb)z^{-1}} \); 
b) \( H(z) = \frac{1 + z^{-1}}{1 + a + jb - (1 - a - jb)z^{-1}} + \frac{1 + z^{-1}}{1 + a - jb - (1 - a + jb)z^{-1}} \); 
c) \( H(z) = 4 \frac{1 - e^{-a\cos(b)}z^{-1}}{1 - 2e^{-a\cos(b)}z^{-1} + e^{-2a}z^{-2}} \); 
d) none of the above.

17. For \( H_c(s) \) from the previous problem, the passband center frequency is approximately at 
\( \Omega_p = b \). Using the bilinear transform and \( T_d = 2.0 \) find \( b \) such that the discrete-time band 
center frequency will be at \( \omega_c = 0.6\pi \). Do not round your result, and enter the second most 
significant digit.

18. Which of the following is an advantage of the bilinear design method as compared to the 
impulse invariance method?

a) lower order filter designs for the same frequency response specifications; 
b) less phase distortion; 
c) it leads to a linear phase filter; 
d) easier design methodology; 
e) introduces no aliasing in the prototype analog filter frequency response; 
f) better suited to the design of high pass filters; 
g) all the above; 
h) none of the above.

19. Someone has proposed using the overlap save method with FFTs to efficiently implement a 
filter designed by the impulse invariance method. Assume the filter is order \( M = 18 \). Which 
of the following is true?

a) This is a good idea since the \( M\log_2N \) efficiency of the FFT will use fewer multiplies than 
the those needed to implement a direct convolution. 
b) This is a good idea because you want to keep your job by efficiently using DSP resources 
in your designs. 
c) This is a bad idea because impulse invariance designs can introduce aliasing. 
d) This is a bad idea because overlap save only works with FIR filters. 
e) (a) and (b); 
f) (c) and (d); 
g) none of the above.

20. Let \( x[n] \) have corresponding complex cepstrum \( \hat{x}[n] \). If \( \hat{x}[n] \) is causal-stable, then \( x[n] \) is:

a) causal-stable; 
b) linear phase; 
c) complex; 
d) real; 
e) non-causal; 
f) even; 
g) minimum phase; 
h) a Hilbert transform pair with \( \hat{x}[n] \).
1. Since \(|A(e^{j\omega})|\) is non-zero at both
   \(\omega = 0\) and \(\omega = \pi\), this is type I.  

2. Alternations at \(\omega = 0, 0.3\pi, 0.5\pi, 0.7\pi + 0.85\pi\).  

3. This is not an extra-ripple case since there is no alternation at \(\omega = \pi\), and so for a high-pass filter there must be exactly \(2+2\) alternations. \(L = 5-2 = 3\)
   \[N = 2 \cdot L + 1 = 7\].  

4. \(W(0.8\pi) = W(0.2\pi)\).  

5. 

6. We see zeroes at \(\omega = 0.15\pi\) and \(0.43\pi\). Due to symmetry of \(|A(e^{j\omega})|\) for a real filter, there must also be zeroes at \(\omega = -0.15\pi, 0.3\pi\) on the unit circle.  

7. \(A = 53\).  

8. \(\beta = 0.1102 (A = 8.7) = 4.882\)  

9. \(\beta = 4.9\)  

10. \(M = \frac{A - 8}{2.285 \Delta \omega} = \frac{53 - 8}{2.285\left(\frac{\pi}{6}\right)} = 100.3\)  

   \[N = \lceil M + 1 \rceil = 102.\] Note that the narrowest transition band was used for \(\Delta \omega\).
9. \( \omega_3 = \frac{3\pi}{4} - \frac{\pi}{32} = 2.25^\circ \) places the ideal transition in the middle of the specified transition band.

10. Need highest sidelobe falloff rate since this is 2 octaves away. Bohman has -24 dB/octave and -96 dB first sideobe, for a total attenuation \( \approx 94 \text{ dB} \) at 2 octaves away, this is much more attenuation than the other windows.

11. You can't do either (a) or (b) because you are just barely meeting spec. in one transition band and in the stopbands.

Reducing \( N \) or increasing \( P \) would cause the first transition to exceed specified width. Decreasing \( P \) would cause ripple level in the stopband to exceed specs.

12. Minimum energy delay corresponds to min. phase, so \( \pi - L \). Maximum energy delay occurs at max. phase (all zeros outside unit circle), so \( \pi - 5 \).

13. Min. phase gives min. group delay.
14. \( H(z) = 2 + 2z^{-1} - 1.5z^{-2} = \frac{z^2 + 2 - 3\frac{1}{2}}{1.5z^2} \)

\[ = \frac{(z - \frac{1}{2})(z + 3\frac{1}{2})}{1.5z^2} \]

Zero at \( z = \frac{1}{2}, -3\frac{1}{2} \)

Double pole at \( z = 0 \)

\[ = 3(1 - \frac{1}{2}z^{-1})(z^{-1} + 3\frac{1}{2}) = 3(1 - \frac{1}{2}z^{-1})(\frac{1}{2}z^{-1} + 3\frac{1}{2}) \]

\( H_{\text{imp}}(z) \) has a zero at \( z = -\frac{3}{2} = -1.5 \)

\( H(z) \)

\( H_{\text{imp}}(z) \)

\( \frac{1}{H_{\text{imp}}(z)} \)

15. Non exhibit symmetry

16. \( H(z) = \frac{1}{1 - \frac{1}{2}z^{-1} + d} + \frac{1}{1 - \frac{1}{2}z^{-1} + d^*} \) for \( d = a + jb \).

\[ = \frac{1 + \frac{1}{2}z^{-1}}{1 + d + (d-1)z^{-1}} + \frac{1 + \frac{1}{2}z^{-1}}{1 + d^* + (d^*-1)z^{-1}} \]

17. \( \phi_0 = b = \frac{2}{\pi} \tan \left( \frac{\mu_0}{2} \right) = \tan \left( \frac{0.672}{2} \right) = 1.376 \)

18.

19.

20.