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EC En 487

Midterm 1

Winter Semester 2006

February 24-28

Open Text Book and Notes.
Time limit: 2.5 hours

Instructor: Brian D. Jeffs

1. Enter all answers on the scantron "bubble" answer sheet.
2. Each problem is worth five points.
3. For numeric answers do *not* round your result. Enter the requested digit (e.g. most significant, or e.g. the 10's digit) into the bubble sheet.
4. Attach all your calculation worksheets to this test and return them to the testing center. Any requests of credit on incorrectly marked answers must be supported by a correct numerical answer on your calculation worksheets.

1. Compute the Z transform of the sequence:
   $x[n] = -\delta[n] - \frac{1}{3}\delta[n-1] - \frac{1}{9}\delta[n-2] - \frac{1}{27}\delta[n-3]$:

   a) $\dfrac{\frac{1}{8}z^{-3} - 1}{1 - \frac{1}{2}z^{-1}}$, $|z| > 0$;   b) $1 + \frac{1}{3}z^{-1} + \frac{1}{9}z^{-2} + \frac{1}{27}z^{-3}$, $|z| > 0$;   c) $\dfrac{\frac{1}{81}z^{-4} - 1}{1 - \frac{1}{3}z^{-1}}$, $|z| > 0$;

   d) $1 + \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2}$, $|z| > 0$;   e) $-\frac{1}{3}^{n} z^{-1}$, $|z| > 0$;   f) none of the above

2. Consider the complex cepstrum $\hat{X}(z) = \ln\{X(z)\}$ where $X(z) = \dfrac{1 - \frac{1}{4}z^{-1}}{(1 + 2z^{-1})(1 - 3z^{-1})}$. Let $\hat{x}(n)$ be the inverse Z transform of $\hat{X}(z)$ obtained using an R.O.C. which includes the unit circle. Which statement correctly describes $\hat{x}(n)$?

   a) causal stable;   b) left sided;   c) two sided;   d) unstable;   e) anti-causal;
   f) real;   g) cannot determine from given information;   h) none of the above.

3. Consider using Cauchey's residue theorem to compute the inverse Z transform of
   $X(z) = \dfrac{(z-2)(z+0.15)(z-\pi)}{(z-\frac{2}{3})(z-e)(z+7)(z+2)}$ for a region of convergence which includes the unit circle. What is the total order of all poles enclosed in the contour when evaluating the inversion integral for $x[-4]$ (i.e. n = - 4) ? Assume the integral is with respect to z, i.e. do not make a substitution of variable for z. For example if one first order, one second order, and one fourth order pole were enclosed, the "total order of all poles" would be 7.
   (Bubble in the numeric answer.)

4. Use Cauchey's residue theorem to compute $x[n]$ for $n = 2$ only, using $X(z)$ and the R.O.C. as specified in question 3 above. $x[2]$ = ?
   (Ignore sign and bubble in the most significant digit of your answer.)

5. Which answer below gives the correct form for $x[n]$, $n > 0$, using $X(z)$ and the R.O.C. as specified in question 3 above? a, b, and c are some constants.

   a) $c\left(\frac{2}{3}\right)^n$;   b) $c\,e^n$;   c) $a + c(0.15)^n$;   d) $a\cos(b\pi n)$;   e) $c\left(\frac{2}{3}\right)^n + b(2)^{-n}$;

   f) $b(7)^n$;   g) none of the above.

6. When computing the inverse of $X(z) = \dfrac{1-3z^{-1}}{(1+\frac{1}{4}z^{-1})(1-2z^{-1})}$, $\frac{1}{4} < |z| < 2$, for $n < 0$, what would be the best integral to evaluate using residue theory?

a) $\dfrac{1}{2\pi j}\oint_{C'} \dfrac{(p-\frac{1}{3})p^{n-1}}{(p+4)(p-\frac{1}{2})}dp$; b) $\dfrac{1}{2\pi j}\oint_{C'} \dfrac{6(p-\frac{1}{3})p^{-n-1}}{(p+4)(p-\frac{1}{2})}dp$; c) $\dfrac{1}{2\pi j}\oint_{C} \dfrac{(z-3)z^{n}}{(z+\frac{1}{4})(z-2)}dz$;

d) $\dfrac{1}{2\pi j}\oint_{C} \dfrac{(1-3z^{-1})z^{n-1}}{(1+\frac{1}{4}z^{-1})(1-2z^{-1})}dz$; e) $\dfrac{1}{2\pi j}\oint_{C'} \dfrac{(p+4)(p-\frac{1}{2})}{(p-\frac{1}{3})p^{n-1}}dp$; f) none of the above.

7. Find the inverse of $X(z)$ from problem 6 for $n < 0$. It will be of the form $x[n] = c\,a^{-n}$, $n < 0$ for some constants a and c. Bubble in the most significant digit of constant c.

8. It is possible to form a Cooley-Tukey fast Fourier transform algorithm to compute a 37 point DFT faster than can be done with the direct DFT sum. Is this statement true (T) or false (F)?

9. Consider Factored N Cooley-Tukey implementation of a 25 point FFT. How many (complex) multiplies must be computed in each butterfly? Do not count multiplies by terms equal to 1, -1, j, or $-j$. Bubble in the least significant digit only, i.e. if the answer is greater than 9, ignore the 10's digit.

10. In the full 25 point Factored N Cooley-Tukey FFT implementation from problem 9, count all butterfly branch and twiddle factor multiplies (even if equal to 1, -1, j, or $-j$). Bubble in the second most significant digit (the 10s digit) for the total number of multiplies.

11. Consider computation of a 256 point DFT. How much faster is a radix 2 FFT implementation than the direct DFT sum? Compute the ratio of the number of (complex) multiplies for the direct DFT over the number of (complex) multiplies in the FFT. Bubble in the most significant digit only. When computing FFT multiplies assume the architecture of text Figure 9.7 (see last exam page), not Figure 9.10, and count all terms of the form W_N^m where m is any exponent.

12. Consider an efficient polyphase implementation of a converter to increase a 24 k samp/sec signal to a 32 k samp/sec rate. Assuming ideal filters, specify the filter corner frequency in radians per sample. Bubble in the most significant digit only.

13. For the sample rate converter of problem 12, how many different polyphase component filters, $e_k[n]$, will there be?

14. For the sample rate converter of problem 12, what must the passband gain of the lowpass filter be to insure no amplitude change in the signal?

15. Consider a polyphase interpolator which upsamples by a factor of 5. Assume the designed lowpass filter coefficients are $h[n] = n+1$ for $0 \le n \le 44$ and zero elsewhere (this is a very bad lowpass filter but we use it for this problem). What is value of $e_3[1]$, i.e. the $n = 1$ tap coefficient in the $k = 3$ polyphase component filter?

16. For your bandpass filter in ECEN 487 lab 2, assume you want a more efficient implementation so you can run a long 523 tap filter at the full 48 k samp/sec and thus separate more closely space signal channels. You plan on using an overlap-save 2048 point FFT based implementation. Assume the FFT architecture of Figure 9.7 in your text and count <u>all</u> multiplies by terms of the form W_N^m where m is any exponent. How many good (usable) samples do you obtain per FFT window?

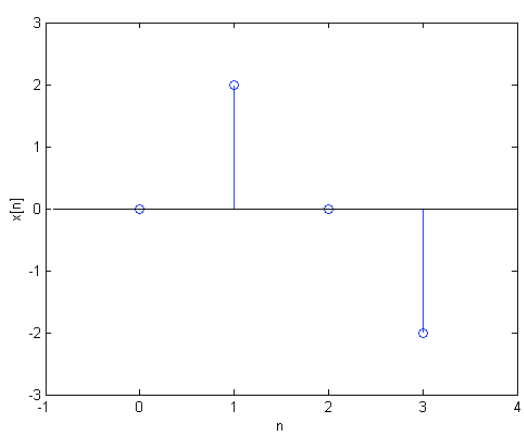
    a) 2048; b) 523; c) 178; d) 1526; e) 1525; f) 511; g) 1733;
    h) none of the above.

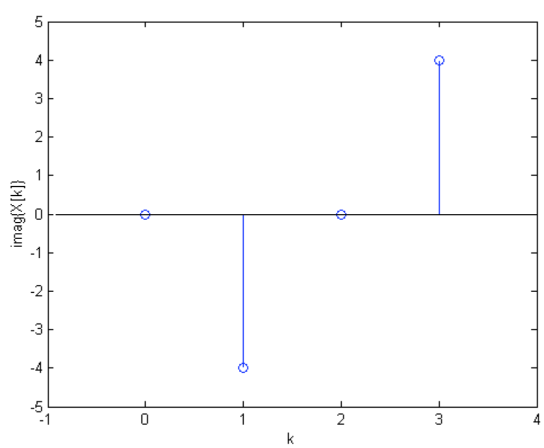
17. Assume (incorrectly) the answer to problem 16 is 1900. Assume all other specifications are the same. How many multiplies must be computed per output sample?

    a) 2048; b) 4.23; c) 107.53; d) 15.3; e) 47.96; f) 24.79; g) 523.

18. Assume (incorrectly) the answer to problem 17 is 65. What is the computational improvement with respect to a direct evaluation of the convolution sum? Compute the ratio of multiplies per output sample for the direct convolution over the multiplies per output for the overlap save implementation. Bubble in the <u>most significant digit</u> of the ratio.

19. $x[n]$ is a 4 sample finite length sequence which is shown below along with the imaginary part of its corresponding 4 point DFT, i.e. imag{ $X[k]$}.





Use a property of the DFT to specify real{ $X[k]$}for $k = 1$. Bubble in its absolute value.

20. What property from text Table 8.2, (see below) best explains how to easily compute real{ $X[k]$} from problem 19? Enter the <u>least significant digit</u> of the table entry number.

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1. $x[n] = -\delta[n] - \frac{1}{3}\delta[n-1] - \frac{1}{9}\delta[n-2] - \frac{1}{27}\delta[n-3]$

$$= \begin{cases} -(1/3)^n & 0 \le n \le 3 \\ 0 & \text{otherwise} \end{cases}$$

$$X(z) = -\sum_{n=0}^{3} (1/3)^n z^{-n} = -\sum_{n=0}^{3} \left(\tfrac{1}{3} z^{-1}\right)^n$$

Geometric series: $\sum_{n=0}^{M-1} a^n = \frac{1-a^M}{1-a}$

so $X(z) = -\frac{1 - \frac{1}{81} z^{-4}}{1 - \frac{1}{3} z^{-1}}$, $|z| > 0$ (no poles) (C)

2. $X(z) = \frac{1 - \frac{1}{4} z^{-1}}{(1+2z^{-1})(1-3z^{-1})}$. Zero at $z = 1/4$, poles at $z = -2, 3$. For $\hat{X}(z)$ the poles and zeros of $X(z)$ both become poles. So $\hat{X}(z)$ has poles at $z = 1/4, -2, 3$. R.O.C. including unit circle is bounded by $z = 1/4$ and $z = -2$ an does not include the origin or $z = \infty$. $\therefore \hat{x}[n]$ is two sided. (C)

3. $x[n] = \sum \text{Res}\{X(z) z^{n-1}\}$ enclosed by C,
$C: |z| = 1$, $n = -4$, one pole of $X(z)$ is interior to C ($z = 2/3$). $z^{n-1} = z^{-4-1} = z^{-5}$ is a fifth order pole at $z = 0$. Total order $= 5 + 1 =$ (6)

4. $x[2] = \left.\dfrac{(2/3-2)(2/3+0.15)(2/3-\pi)(2/3)^{n-1}}{(2/3-e)(2/3+7)(2/3+2)}\right|_{n=2} = -42.83\times10^{-3}$ (4)

5. all terms in 4 above are constant except $(2/3)^{n-1} = \frac{3}{2}(2/3)^n$ so (a) is the correct form. $x[n] = -64.25\times10^{-3}(2/3)^n$

6. $x[n] = \dfrac{1}{2\pi j}\oint_{c'} X(1/p)\,p^{n-1}\,dp$, $c' = \{p : 1/p \in C\}$

$$= \frac{1}{2\pi j}\oint_{c'} \frac{(1-3p)p^{n-1}}{(1+1/4p)(1-2p)}\,dp$$

$$= \frac{1}{2\pi j}\oint_{c'} \frac{6(p-1/3)\,p^{-n-1}}{(p+4)(p-1/2)}\,dp$$ (b)

7. $x[n] = \left.\text{Res}\left\{\dfrac{6(p-1/3)p^{-n-1}}{(p+4)(p-1/2)}\right\}\right|_{p=1/2}$

$$= \frac{6(1/2-1/3)(1/2)^{-n-1}}{(1/2+4)} = \frac{4}{9}(1/2)^{-n}$$

$c = 4/9 = 0.444$ (4)

8. 37 is prime, so we cannot factor and thus decimate $x[n]$ (F)

9. W_5^{kn} $0 \le k,n \le 4$ is equal to 1 when either k or n = 0. Of the 25 butterfly branches, 9 correspond to k and/or n = 0, leaving 16 multiplies. $W_5^{kn} \ne -1, j$ or $-j$ for any k, n. (6)

10. 25 X's per butterfly, 10 butterflies. One column of 25 twiddle factors in the middle. $(10)(25)+25=275$ X's. (7)

11. DFT X's; $N^2 = (256)^2 = 65536$
FFT X's; $N\log_2 N = (256)(8) = 2048$
$\frac{N^2}{N\log_2 N} = \frac{N}{\log_2 N} = \frac{256}{8} = 32$ (3)

12. $\frac{L}{M} = \frac{32}{24} = \frac{4}{3}$. $\omega_c = \min\left(\frac{\pi}{4}, \frac{\pi}{3}\right) = \frac{\pi}{4}$.
$\frac{\pi}{4} = 0.7854$ (7)

13. $L = 4$, 4 polyphase filters (4)

14. $G = L = 4$ (4)

15. $e_k[n] = h[nL+k]$
$e_3[1] = h[L+3] = (5+3)+1 =$ (9)

16. $N_g = L - P + 1 = 2048 - 523 + 1 = 1526$ (d)

17. X's / window $= 2L\log_2 L + L$
$= 2(2048)(11) + 2048 = 47104$
X's / output samp $= 47104/1900 = 24.79$ (f)

18. Convolution; P X's/output = 523
$523/65 = 8.05$ (8)

19. $x[n]$ is odd so $X[k]=0$, (0)

20. Table entry 17 (or 14) or (3, 4, 6) (7)