Due Dates

This is a four week lab. All TA check off must be completed prior to the specified lab book write-up submission time or the lab will be marked late.

Submit answers to the questions from the last page of this handout at the beginning of lab class on:
Tues., Mar. 27.

Lab book write-up copy submission, by 4:00 p.m. in Dr. Jeffs’ box outside his office door:
All sections, Wed. April 11.

Objectives

The purpose of this lab is for each student to build a working real-time acoustic direction finder. The processor will compute the phase between narrowband signals seen at two microphones using a sample autocorrelation at complex baseband. Given this phase, and the known baseline geometry between the microphones, a direction of arrival will be computed.

Reading Assignment


2. The Introduction Section below, in the lab handout.

Introduction

In many applications, including for example radio astronomy, wireless communications, SONAR detection, intelligence signal interception, autonomous robotic vehicle navigation, and medical ultrasound imaging, it is desirable to estimate the direction of arrival (DOA) of the wavefront emitted from an acoustic or electromagnetic source. Figure 1 illustrates a simple two sensor DOA estimation configuration for determining the direction to a narrowband acoustic source. Let $x_1(t)$ and $x_0(t)$ represent the continuous-time signals received by two microphones separated by distance $d$. Due to the different propagation path distances to the two sensors, the signal at $x_1(t)$ (neglecting noise) is delayed and attenuated relative to $x_0(t)$ such that...
\[ x_1(t) = \alpha x_0(t-\Delta), \quad \text{where} \ \Delta = (d/c) \sin \theta, \quad (1) \]
where \( \alpha \) is a positive real attenuation, and \( c = 344 \text{ m/s} \) is the speed of sound in air at 71 °F. We wish to estimate \( \theta \).

Narrowband signals (i.e. where the signal bandwidth is less than 10\% of the signal center frequency, \( \Omega_c \)) can be conveniently represented using complex envelope notation, i.e. 
\[ x_0(t) = \text{Re}\{e^{j\Omega_c t} s_0(t)\}, \]
where \( s_0(t) \) is the lowpass complex baseband envelope signal. In a communications environment, you may think of \( e^{j\Omega_c t} \) as a "carrier" signal (as in a radio frequency carrier in a broadcast transmission) and \( s_0(t) \) as the "modulating" or information bearing signal.

Because of the narrowband assumption, time delay \( \Delta \) acts approximately as a constant phase shift across the signal band, so we may write 
\[ x_1(t) = \alpha x_0(t - \Delta) = \alpha \text{Re}\{e^{j\Omega_c t} [e^{-j\Omega_c \Delta} s_0(t)]\}. \]
Equality holds for pure sinusoids.

After sampling we have,
\[ x_0[n] = \text{Re}\{e^{j\omega_c n} s_0[n]\}, \quad \text{and} \quad x_1[n] = \text{Re}\{e^{j\omega_c n} s_1[n]\} = \alpha \text{Re}\{e^{j\Omega_c n} (e^{-j\Omega_c \Delta} s_0[n])\}, \quad (2) \]
where \( \omega_c = 2\pi \frac{\Omega_c}{\Omega_s} \), and \( \Omega_s \) is the sample frequency. Now \( s_0[n] \) is the baseband modulating signal, and \( e^{j\omega_c n} \) is the carrier. This is the same Hilbert transform relationship for bandpass

Figure 1. Direction finder signal geometry.
signals discussed in Section 12.4.2 of your text (with \( x_0[n] \) here corresponding to \( s_r[n] \) in the book, and \( s_0[n] \) here corresponding to \( x[n] \)).

For phase estimation it is more convenient to work with the complex baseband envelope signals, \( s_0[n] \) and \( s_1[n] \), which can be computed with a Hilbert transform followed by a complex bandshift as shown in Figure 2.

\[
x_0[n] \quad \text{Hilbert Xfm} \quad \begin{cases} \text{Re}\{X_0[n]\} \\ \text{Im}\{X_0[n]\} \end{cases} \quad X_0[n] \quad \text{Complex Multiply} \quad s_0[n] = e^{-j\omega c n} \]

Figure 2. Baseband complex envelope signal generation.

Note that a Hilbert transform is merely a 90° phase shifter allpass filter and that

\[
X_0[n] = x_0[n] + j(\text{Hilbert}\{x_0[n]\}) = \text{Re}\{e^{j\omega_c n}s_0[n]\} + j\text{Im}\{e^{j\omega_c n}s_0[n]\} = e^{j\omega_c n}s_0[n].
\]

Consider the inner product of the two baseband microphone signals. Using (2) we find

\[
\begin{align*}
\langle s_0[n], s_1[n] \rangle^* &= e^{j\Omega_c \Delta} |s_0[n]|^2, \\
\angle \langle s_0[n], s_1[n] \rangle^* &= \Omega_c \Delta, \text{ so using (1),} \\
\theta &= \sin^{-1} \left( \frac{c}{d\Omega_c} \angle \langle s_0[n], s_1[n] \rangle^* \right),
\end{align*}
\]

where * indicates complex conjugate, and \( \angle \) is the phase angle of a complex number. Clearly we can compute the DOA directly from the baseband envelope inner product.

We can improve on Figure 2 and Equation (3) by including averaging to reduce the effects of noise, and replacing the Hilbert transform with the equivalent structure shown (once for each mic signal) in Figure 3. The advantage of this “complex baseband bandshifter” approach is that the lowpass filters reject out-of-band noise and interference, and the filters can use decimating polyphase implementation for efficiency, where the Hilbert transform filter must operate at the highest sample frequency.
The conjugate, complex product, and summation blocks are really performing a crosscorrelation function as described in the textbook, Appendix A.2 and A.3. The summation should be over approximately 0.1 to 5 seconds of samples.

**Experiment 1 Construct a real-time DSP acoustic direction finder.**

**Procedure**

1. Write a MATLAB DSP code to implement the direction finder algorithm illustrated in Figure 3. Use your Lab 3 code as the starting point. You will need some elements from your Lab 3 code (e.g. complex multiplies, building a sine - cosine table).

2. Design a suitable FIR lowpass filter for your system.

3. Use a function generator and an amplified speaker as a narrowband signal source. Select a transmit frequency and sample frequency to be compatible.

4. Note that careful choice of these frequency relationships can significantly reduce the required length of your sine - cosine table. Why?
5. Use a stereo microphone mixer / preamp and two microphones as input to your direction finder. Mount the microphones a known distance, $d$, apart that is approximately 1/2 wavelength at your transmit frequency.

6. Adjust mic separation, frequencies, and/or scale factors to calibrate the angle estimation for accuracy.

7. Demonstrate proper direction finding operation to the TA and have her/him sign off completion in your lab book.

**Experiment 2  DOA estimation competition.**

**Procedure**

1. The final day of class we will hold a competition for all working direction finders. The professor will choose the source location. Each team will mount their microphones on the lecture table in turn.

2. Performance awards will be given for two categories: 1) lowest average angle estimation error with high SNR, and 2) lowest source power level for a fixed angle error tolerance level.

3. Awards for the two winning teams will be of the edible variety.

**Conclusions**

Write a paragraph or two of conclusions for your lab experience. Discuss any additional implications of what you observed. What other applications can you see for using a direction finder? What improvements could be made to the basic design to make it more usable? Describe what you feel are the important principals demonstrated in this lab, and note anything that you learned unexpectedly. What debug and redesign procedures did you need to perform to get it to work?
Questions (Due at beginning of second lab session)

1. Mathematically prove the equivalence (to within a scale factor) of the two approaches for complex baseband signal generation as shown in Figures 2 and 3.

2. Describe how you could use the MATLAB function atan2. Why not use atan?

3. What is the highest stopband corner frequency your lowpass filters of Figure 3 can be designed for without destroying the equivalence you showed in question 1? What is the lowest practical passband corner frequency (make some assumption about the narrowband signal).

4. Explain what problems occurs if \( d \) is greater than about 1/2 wavelength at the center frequency.
Question solutions:

1. For Figure 2:
   \[
   x_0[n] = \text{Re}\{e^{j\omega_c n} s_0[n]\},
   \]
   \[
   X_0[n] = e^{j\omega_c n} s_0[n] \quad \text{(by Hilbert transform definition)},
   \]
   \[
   e^{-j\omega_c n} X_0[n] = e^{-j\omega_c n} e^{j\omega_c n} s_0[n],
   \]
   \[
   = s_0[n].
   \]

For Figure 3:
   \[\]
   \[
   x_0[n] = \text{Re}\{e^{j\omega_c n} s_0[n]\},
   \]
   \[
   = (\cos \omega_c n) \text{Re}\{s_0[n]\} - (\sin \omega_c n) \text{Im}\{s_0[n]\},
   \]
   \[
   X_{0,1}[n] = (\cos^2 \omega_c n) \text{Re}\{s_0[n]\} - (\cos \omega_c n \sin \omega_c n) \text{Im}\{s_0[n]\},
   \]
   \[
   = \frac{1}{2} (1 + \cos 2\omega_c n) \text{Re}\{s_0[n]\} - \frac{1}{2} (\sin 2\omega_c n) \text{Im}\{s_0[n]\},
   \]
   \[
   X_{0,0}[n] = -(\sin \omega_c n \cos \omega_c n) \text{Re}\{s_0[n]\} + (\sin^2 \omega_c n) \text{Im}\{s_0[n]\},
   \]
   \[
   = \frac{1}{2} (\sin 2\omega_c n) \text{Re}\{s_0[n]\} + \frac{1}{2} (1 - \cos 2\omega_c n) \text{Im}\{s_0[n]\}.
   \]

Now, lowpass filter \(X_{0,1}[n]\) and \(X_{0,0}[n]\), which removes all the double frequency product terms, so
   \[
   s_{0,1}[n] = \frac{1}{2} \text{Re}\{s_0[n]\},
   \]
   \[
   s_{0,0}[n] = \frac{1}{2} \text{Im}\{s_0[n]\},
   \]
   \[
   s_0[n] = s_{0,1}[n] + j s_{0,0}[n] = \frac{1}{2} \text{Re}\{s_0[n]\} + j \frac{1}{2} \text{Im}\{s_0[n]\},
   \]
   \[
   = \frac{1}{2} s_0[n].
   \]

Thus the two systems are equivalent to within a scale factor of 1/2.

2. \(\text{atan2}\) is preferred over \(\text{atan}\) because it is a two argument inverse tangent where you specify both the numerator and denominator of a ratio (e.g. numerator is imaginary part and denominator is real part of a complex number you wish to compute phase for.) \(\text{atan2}\) can return angles in the range \(-\pi\) to \(\pi\), while \(\text{atanf}\) only covers \(-\pi/2\) to \(\pi/2\), which is the period of the arc tangent function.

3. As seen in question one above, assuming \(s_0[n]\) is very narrowband, the filter cutoff must be below \(2\omega_c\). Assuming a full complex bandwidth of 50 Hz for \(s_0[n]\), the lowest cutoff frequency would be 25 Hz, because at baseband the bandwidth of \(s_0[n]\) is centered about the origin, d.c.
4. If $d$ is greater than 1/2 wavelength then spatial aliasing occurs, and two or more distinct arrival angles, $\theta$, would produce the same phase shift across the array. This produces an ambiguity in direction finding.